BOSTES
BOARD OF STUDIES TEACHING & EDUCATIONAL STANDARDS NSW

MATHEMATICS EXTENSION 1 STAGE 6

DRAFT SYLLABUS FOR CONSULTATION

20 JULY – 31 AUGUST 2016

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THE BOSTES SYLLABUS DEVELOPMENT PROCESS

BOSTES began its syllabus development process for Stage 6 English, Mathematics, Science and History in 2014. This followed state and territory Education Ministers' endorsement of senior secondary Australian curriculum.

The development of the Stage 6 syllabuses involved expert writers and opportunities for consultation with teachers and other interest groups across NSW in order to receive the highest-quality advice across the education community.

A number of key matters at consultations were raised, including the need for the curriculum to cater for the diversity of learners, the broad range of students undertaking Stage 6 study in NSW, development of skills and capabilities for the future, school-based assessment and providing opportunities for assessing and reporting student achievement relevant for post-school pathways.

There was broad support that changes to curriculum and assessment would contribute to the reduction of student stress. BOSTES will continue to use NSW credentialling processes aligned with Stage 6 assessment and HSC examination structures.

A summary of the BOSTES syllabus development process is available at http://www.boardofstudies.nsw.edu.au/syllabuses/syllabus-development.

ASSISTING RESPONDENTS

The following icons are used to assist respondents:

for your information	This icon indicates general information that assists in reading or understanding the information contained in the document. Text introduced by this icon will not appear in the final syllabus.
consult	This icon indicates material on which responses and views are sought through consultation.

CONSULTATION

The *Mathematics Extension 1 Stage 6 Draft Syllabus* is accompanied by an online consultation survey on the BOSTES website. The purpose of the survey is to obtain detailed comments from individuals and systems/organisations on the syllabus. Please comment on both the strengths and the weaknesses of the draft syllabus. Feedback will be considered when the draft syllabus is revised.

The consultation period is from 20 July to 31 August 2016.

Written responses may be forwarded to:

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INTRODUCTION

STAGE 6 CURRICULUM

Board of Studies, Teaching and Educational Standards NSW (BOSTES) Stage 6 syllabuses have been developed to provide students with opportunities to further develop skills which will assist in the next stage of their lives, whether that is academic study, vocational education or employment.
The purpose of the Higher School Certificate program of study is to:

- provide a curriculum structure which encourages students to complete secondary education
- foster the intellectual, social and moral development of students, in particular developing their:
  - knowledge, skills, understanding, values and attitudes in the fields of study they choose
  - capacity to manage their own learning
  - desire to continue learning in formal or informal settings after school
  - capacity to work together with others
  - respect for the cultural diversity of Australian society
- provide a flexible structure within which students can prepare for:
  - further education and training
  - employment
  - full and active participation as citizens
- provide formal assessment and certification of students' achievements
- provide a context within which schools also have the opportunity to foster students' physical and spiritual development.

The Stage 6 syllabuses reflect the principles of the BOSTES *K–10 Curriculum Framework* and *Statement of Equity Principles*, and the *Melbourne Declaration on Educational Goals for Young Australians* (December 2008). The syllabuses build on the continuum of learning developed in the K–10 syllabuses.

The Stage 6 syllabuses provide a set of broad learning outcomes that summarise the knowledge, understanding, skills, values and attitudes essential for students to succeed in and beyond their schooling. In particular, the literacy and numeracy skills needed for future study, employment and life are provided in Stage 6 syllabuses in alignment with the *Australian Core Skills Framework (ACSF)*.

The syllabuses have considered agreed Australian curriculum content and included content that clarifies the scope and depth of learning in each subject.

Stage 6 syllabuses support a standards-referenced approach to assessment by detailing the essential knowledge, understanding, skills, values and attitudes students will develop and outlining clear standards of what students are expected to know and be able to do. In accordance with the *Statement of Equity Principles*, Stage 6 syllabuses take into account the diverse needs of all students. The syllabuses provide structures and processes by which teachers can provide continuity of study for all students.

DIVERSITY OF LEARNERS

NSW Stage 6 syllabuses are inclusive of the learning needs of all students. Syllabuses accommodate teaching approaches that support student diversity including Students with special education needs, Gifted and talented students and Students learning English as an additional language or dialect (EAL/D).

STUDENTS WITH SPECIAL EDUCATION NEEDS

All students are entitled to participate in and progress through the curriculum. Schools are required to provide additional support or adjustments to teaching, learning and assessment activities for some students. Adjustments are measures or actions taken in relation to teaching, learning and assessment that enable a student to access syllabus outcomes and content and demonstrate achievement of outcomes.

Students with special education needs can access the Stage 6 outcomes and content in a range of ways. Students may engage with:

- syllabus outcomes and content with adjustments to teaching, learning and/or assessment activities
- selected outcomes and content appropriate to their learning needs
- selected Stage 6 Life Skills outcomes and content appropriate to their learning needs.

Decisions regarding adjustments should be made in the context of collaborative curriculum planning with the student, parent/carer and other significant individuals to ensure that syllabus outcomes and content reflect the learning needs and priorities of individual students.

Further information can be found in support materials for:

- Mathematics
- Special education needs
- Life Skills.

GIFTED AND TALENTED STUDENTS

Gifted students have specific learning needs that may require adjustments to the pace, level and content of the curriculum. Differentiated educational opportunities assist in meeting the needs of gifted students.

Generally, gifted students demonstrate the following characteristics:

- the capacity to learn at faster rates
- the capacity to find and solve problems
- the capacity to make connections and manipulate abstract ideas.

There are different kinds and levels of giftedness. Gifted and talented students may also possess learning difficulties and/or disabilities that should be addressed when planning appropriate teaching, learning and assessment activities.

Curriculum strategies for gifted and talented students may include:

- differentiation: modifying the pace, level and content of teaching, learning and assessment activities
- acceleration: promoting a student to a level of study beyond their age group
- curriculum compacting: assessing a student’s current level of learning and addressing aspects of the curriculum that have not yet been mastered.

School decisions about appropriate strategies are generally collaborative and involve teachers, parents and students with reference to documents and advice available from BOSTES and the education sectors.

Gifted and talented students may also benefit from individual planning to determine the curriculum options, as well as teaching, learning and assessment strategies, most suited to their needs and abilities.

STUDENTS LEARNING ENGLISH AS AN ADDITIONAL LANGUAGE OR DIALECT (EAL/D)

Many students in Australian schools are learning English as an additional language or dialect (EAL/D). EAL/D students are those whose first language is a language or dialect other than Standard Australian English and who require additional support to assist them to develop English language proficiency.

EAL/D students come from diverse backgrounds and may include:

- overseas and Australian-born students whose first language is a language other than English, including creoles and related varieties
- Aboriginal and Torres Strait Islander students whose first language is Aboriginal English, including Kriol and related varieties.

EAL/D students enter Australian schools at different ages and stages of schooling and at different stages of Standard Australian English. They have diverse talents and capabilities and a range of prior learning experiences and levels of literacy in their first language and in English. EAL/D students represent a significant and growing percentage of learners in NSW schools. For some, school is the only place they use English.

EAL/D students are simultaneously learning a new language and the knowledge, understanding and skills of the Mathematics Extension 1 Stage 6 syllabus through that new language. They require additional time and support, along with informed teaching that explicitly addresses their language needs, and assessments that take into account their developing language proficiency.

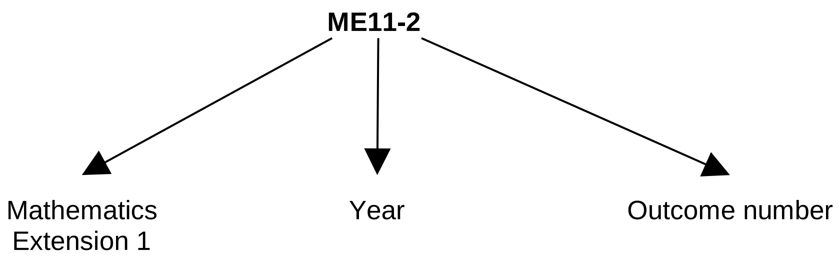
MATHEMATICS EXTENSION 1 KEY

The following codes and icons are used in the *Mathematics Extension 1 Stage 6 Draft Syllabus*.

OUTCOME CODING

Syllabus outcomes have been coded in a consistent way. The code identifies the subject, Year and outcome number.

In the *Mathematics Extension 1 Stage 6 Draft Syllabus*, outcome codes indicate the subject, Year and outcome number. For example:

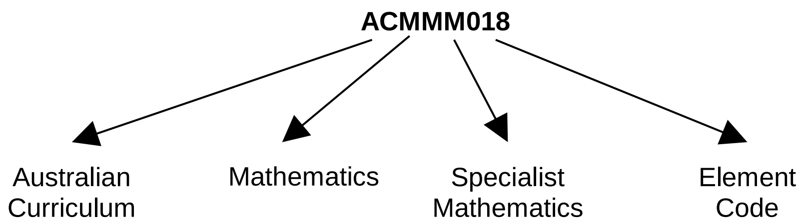


Outcome code	**Interpretation**
ME11-1	Mathematics Extension, Year 11 – Outcome number 1
ME12-4	Mathematics Extension 1, Year 12 – Outcome number 4

CODING OF AUSTRALIAN CURRICULUM CONTENT

Australian curriculum content descriptions included in the syllabus are identified by an Australian curriculum code which appears in brackets at the end of each content description, for example:

Factorise cubic polynomials in cases where a linear factor is easily obtained (ACMMM018).



Where a number of content descriptions are jointly represented, all description codes are included, eg (ACMMM001, ACMGM002, ACMSM003).

CODING OF LEARNING OPPORTUNITIES

The syllabus provides opportunities for modelling applications and exploratory work. These should enable candidates to make connections and appreciate the use of mathematics and appropriate digital technology.

M	This identifies an opportunity for explicit modelling applications or investigations that may or may not involve real-life applications or cross-strand integration.
E	This identifies opportunities for extended exploratory work. Such opportunities allow students to investigate in a rich way the evolution of mathematics or mathematics in practice. Opportunities such as this could form the basis of an internal, non-examination based assessment and are outside the scope of the HSC examination.

As these opportunities are both an integral part of each strand and merge strands together, they are identified by the letter at the end of the relevant content description.

For example: apply knowledge of graphical relationships to solve problems in practical and abstract contexts **M**

LEARNING ACROSS THE CURRICULUM ICONS

Learning across the curriculum content, including cross-curriculum priorities, general capabilities and other areas identified as important learning for all students, is incorporated and identified by icons in the *Mathematics Extension 1 Stage 6 Syllabus*.

Cross-curriculum priorities	
	Aboriginal and Torres Strait Islander histories and cultures
	Asia and Australia's engagement with Asia
	Sustainability
General capabilities	
	Critical and creative thinking
	Ethical understanding
	Information and communication technology capability
	Intercultural understanding
	Literacy
	Numeracy
	Personal and social capability
Other learning across the curriculum areas	
	Civics and citizenship
	Difference and diversity
	Work and enterprise

RATIONALE

for your information

The rationale describes the distinctive nature of the subject and outlines its relationship to the contemporary world and current practice. It explains the place and purpose of the subject in the curriculum, including:

- why the subject exists
- the theoretical underpinnings
- what makes the subject distinctive
- why students would study the subject
- how it contributes to the purpose of the Stage 6 curriculum
- how it prepares students for post-school pathways.

consult

Mathematics is the study of order, relation, pattern, uncertainty and generality and is underpinned by observation, logical reasoning and deduction. From its origin in counting and measuring, its development throughout history has been catalysed by its utility in explaining real-world phenomena and its inherent beauty. It has evolved in highly sophisticated ways to become the language now used to describe many aspects of the modern world.

Mathematics is an interconnected subject that involves understanding and reasoning about concepts and the relationships between those concepts. It provides a framework for thinking and a means of communication that is powerful, logical, concise and precise.

The Mathematics Stage 6 syllabuses are designed to offer opportunities for students to think mathematically. Mathematical thinking is supported by an atmosphere of questioning, communicating, reasoning and reflecting and is engendered by opportunities to generalise, challenge, find connections and to think critically and creatively.

All Mathematics Stage 6 syllabuses provide opportunities to develop students' 21st-century knowledge, skills, understanding, values and attitudes. As part of this, in all courses students are encouraged to learn with the use of appropriate technology and make appropriate choices when selecting technologies as a support for mathematical activity.

Mathematics Extension 1 is focused on enabling students to develop a thorough understanding of and competence in further aspects of mathematics. The course provides opportunities to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Students of Mathematics Extension 1 will be able to develop an appreciation of the interconnected nature of mathematics, its beauty and its functionality.

Mathematics Extension 1 provides a basis for progression to further study in mathematics or related disciplines in which mathematics has a vital role at a tertiary level. An understanding and exploration of Mathematics Extension 1 is also advantageous for further studies in such areas as science, engineering, finance and economics.

THE PLACE OF THE MATHEMATICS EXTENSION 1 STAGE 6 DRAFT SYLLABUS IN THE K–12 CURRICULUM

for your information

NSW syllabuses include a diagram that illustrates how the syllabus relates to the learning pathways in K–12. This section places the Mathematics Extension 1 Stage 6 syllabus in the K–12 curriculum as a whole.

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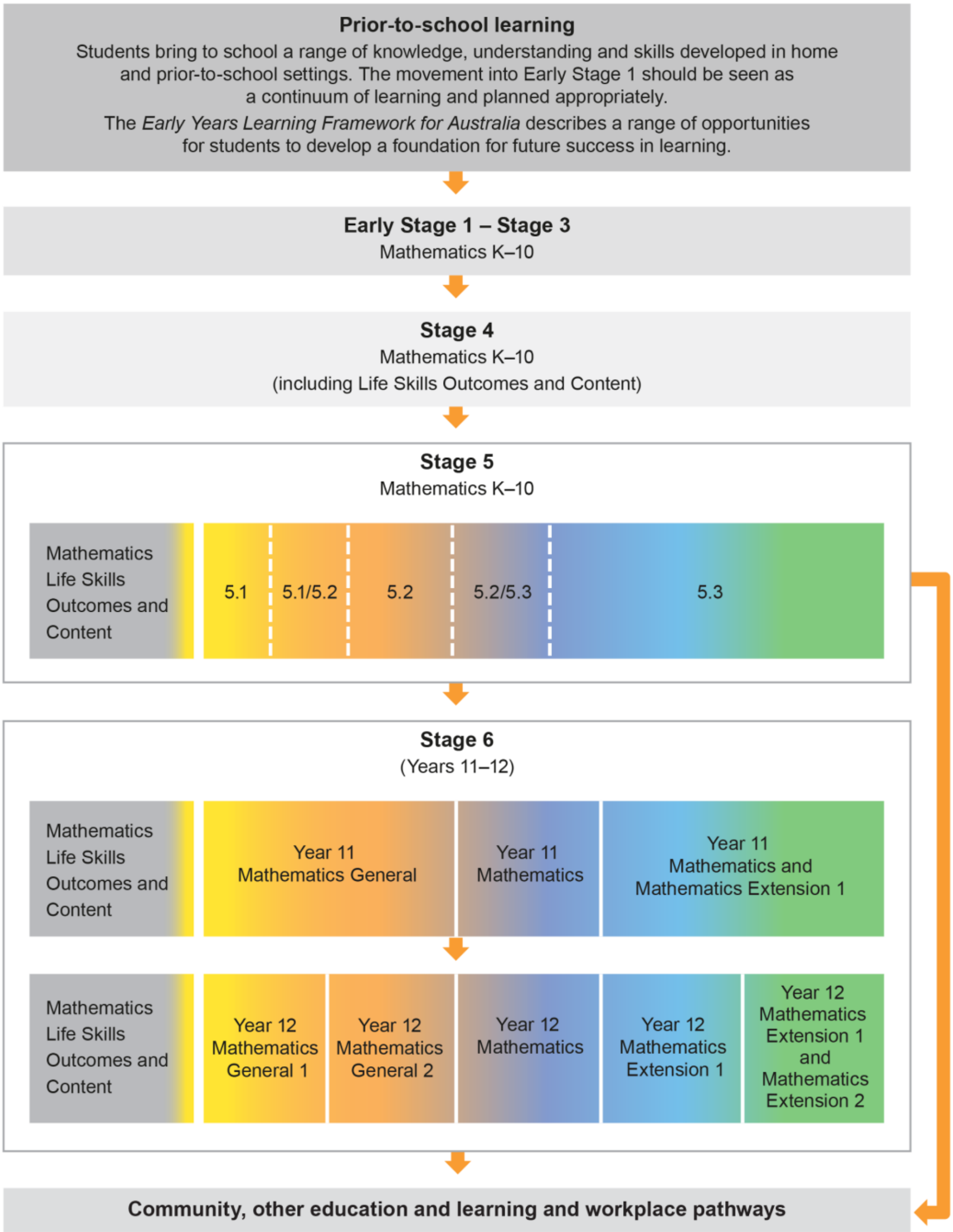

AIM

for your information

In NSW syllabuses, the aim provides a succinct statement of the overall purpose of the syllabus. It indicates the general educational benefits for students from programs based on the syllabus.

The aim, objectives, outcomes and content of a syllabus are clearly linked and sequentially amplify details of the intention of the syllabus.

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The study of Mathematics Extension 1 in Stage 6 enables students to extend their knowledge and understanding of what it means to work mathematically, develop their skills to reason logically, generalise, make connections, and enhance their understanding of how to communicate in a concise and systematic manner.

OBJECTIVES

for your information

In NSW syllabuses, objectives provide specific statements of the intention of a syllabus. They amplify the aim and provide direction to teachers on the teaching and learning process emerging from the syllabus. They define, in broad terms, the knowledge, understanding, skills, values and attitudes to be developed through study in the subject. They act as organisers for the intended outcomes.

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KNOWLEDGE, UNDERSTANDING AND SKILLS

Students:

- develop efficient strategies to solve problems using pattern recognition, generalisation and modelling techniques
- develop the ability to use concepts and skills and apply complex techniques to the solution of problems in the areas of trigonometry, functions and modelling, calculus and statistical analysis
- use technology effectively and apply critical thinking to recognise appropriate times for such use
- develop the ability to interpret, justify and communicate mathematics in a variety of forms

VALUES AND ATTITUDES

Students will value and appreciate:

- mathematics as an essential and relevant part of life, recognising that its development and use has been largely in response to human needs by societies all around the globe
- the importance of resilience and self-motivation in undertaking mathematical challenges and taking responsibility for their own learning and evaluation of their mathematical development

OUTCOMES

for your information

In NSW syllabuses, outcomes provide detail about what students are expected to achieve at the end of each Year in relation to the objectives. They indicate the knowledge, understanding and skills expected to be gained by most students as a result of effective teaching and learning. They are derived from the objectives of the syllabus.

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TABLE OF OBJECTIVES AND OUTCOMES – CONTINUUM OF LEARNING

Objective Students: ● develop efficient strategies to solve problems using pattern recognition, generalisation and modelling techniques	
Year 11 Mathematics Extension 1 outcomes A student:	**Year 12 Mathematics Extension 1 outcomes** A student:
ME11-1 uses algebraic and graphical concepts in the solutions and sketching of functions and their inverses	
	ME12-1 applies calculus techniques to model and solve problems

Objective Students: ● develop the ability to use concepts and skills and apply complex techniques to the solution of problems in the areas of trigonometry, functions and modelling, calculus and statistical analysis	
Year 11 Mathematics Extension 1outcomes A student:	**Year 12 Mathematics Extension 1 outcomes** A student:
ME11-2 applies concepts and techniques of inverse trigonometric functions in the solutions of problems	**ME12-2** applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations

ME11-3 applies algebraic techniques to find solutions of equations and inequalities	**ME12-3** manipulates algebraic expressions and graphical functions to solve problems involving remainder and factor theorems and finding roots of polynomials
ME11-4 applies their understanding of the concept of a derivative to solve related rates of change problems	**ME12-4** uses integral calculus in the solution of problems including volumes of solids of revolution requiring identification and use of appropriate substitutions, partial fractions and integration by parts
ME11-5 uses concepts of permutations and combinations to solve problems involving counting or ordering	**ME12-5** applies appropriate statistical processes to present, analyse and interpret data

Objective Students: ● use technology effectively and apply critical thinking to recognise appropriate times for such use	
Year 11 Mathematics Extension 1 outcomes A student:	**Year 12 Mathematics Extension 1 outcomes** A student:
ME11-6 uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts	**ME12-6** chooses and uses appropriate technology to solve problems in a range of contexts, and applies critical thinking to recognise appropriate times for such use

Objective Students: ● develop the ability to interpret, justify and communicate mathematics in a variety of forms	
Year 11 Mathematics Extension 1 outcomes A student:	**Year 12 Mathematics Extension 1 outcomes** A student:
ME11-7 communicates making comprehensive use of mathematical language, notation, diagrams and graphs	**ME12-7** evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

COURSE STRUCTURE AND REQUIREMENTS

for your information

The following provides an outline of the Year 11 and Year 12 course structure and requirements for the Mathematics Extension 1 Stage 6 Draft Syllabus with indicative hours, arrangement of content, and an overview of course content.

consult

	Mathematics Extension	**Indicative hours**
Year 11 course (60 hours)	The course is organised into strands, with the strands divided into topics.	
	Trigonometric Functions	15
	Functions	15
	Calculus	15
	Statistical Analysis	15
	Modelling and applications are an integral part of each strand and merge strands together, enabling candidates to make connections and appreciate the use of mathematics and appropriate technology.	
Year 12 course 60 hours)	The course is organised into strands, with the strands divided into topics.	
	Trigonometric Functions	20
	Calculus	25
	Statistical Analysis	15
	Modelling and applications are an integral part of each strand and merge strands together, enabling candidates to make connections and appreciate the use of mathematics and appropriate technology.	

For the Year 11 course:

- the Year 11 Mathematics course should be taught prior to or concurrently with this course.
- 60 indicative hours are required to complete the course
- students should experience content in the course in familiar and routine situations as well as unfamiliar or contextual situations.
- students should be provided with regular opportunities involving the integration of technology to enrich the learning experience.

For the Year 12 course:

- the Year 12 Mathematics course should be taught prior to or concurrently with this course.
- the Preliminary course is a prerequisite
- 60 indicative hours are required to complete the course
- students should experience content in the course in familiar and routine situations as well as unfamiliar or contextual situations.
- students should be provided with regular opportunities involving the integration of technology to enrich the learning experience.

ASSESSMENT

for your information

The key purpose of assessment is to gather valid and useful information about student learning and achievement. It is an essential component of the teaching and learning cycle. School-based assessment provides opportunities to measure student achievement of outcomes in a more diverse way than the HSC examination.

BOSTES continues to promote a standards-referenced approach to assessing and reporting student achievement. Assessment for, as and of learning are important to guide future teaching and learning opportunities and to give students ongoing feedback. These approaches are used individually or together, formally or informally, to gather evidence of student achievement against standards. Assessment provides teachers with the information needed to make judgements about students' achievement of outcomes.

Ongoing stakeholder feedback, analysis of BOSTES examination data and information gathered about assessment practices in schools has indicated that school-based and external assessment requirements require review and clarification. The HSC Reforms outline changes to school-based and HSC assessment practices to:

- make assessment more manageable for students, teachers and schools
- maintain rigorous standards
- strengthen opportunities for deeper learning
- provide opportunities for students to respond to unseen questions, and apply knowledge, understanding and skills to encourage in-depth analysis
- support teachers to make consistent judgements about student achievement.

Students with special education needs
Some students with special education needs will require adjustments to assessment practices in order to demonstrate what they know and can do in relation to syllabus outcomes and content. The type of adjustments and support will vary according to the particular needs of the student and the requirements of the assessment activity. Schools can make decisions to offer adjustments to coursework and school-based assessment.

Life Skills
Students undertaking Years 11–12 Life Skills courses will study selected outcomes and content. Assessment activities should provide opportunities for students to demonstrate achievement in relation to the outcomes, and to apply their knowledge, understanding and skills to a range of situations or environments.

The following general descriptions have been provided for consistency. Further advice about assessment, including in support materials, will provide greater detail.

Assessment for Learning	• enables teachers to use formal and informal assessment activities to gather evidence of how well students are learning • teachers provide feedback to students to improve their learning • evidence gathered can inform the directions for teaching and learning programs.
Assessment as Learning	• occurs when students use self-assessment, peer-assessment and formal and informal teacher feedback to monitor and reflect on their own learning, consolidate their understanding and work towards learning goals.
Assessment of Learning	• assists teachers to use evidence of student learning to assess student achievement against syllabus outcomes and standards at defined key points within a Year or Stage of learning.
Formal assessment	• tasks which students undertake as part of the internal assessment program, for example a written examination, research task, oral presentation, performance or other practical task • tasks appear in an assessment schedule and students are provided with sufficient written notification • evidence is gathered by teachers to report on student achievement in relation to syllabus outcomes and standards, and may also be used for grading or ranking purposes.
Informal assessment	• activities undertaken and anecdotal evidence gathered by the teacher throughout the learning process in a less prescribed manner, for example class discussion, questioning and observation • used as part of the ongoing teaching and learning process to gather evidence and provide feedback to students • can identify student strengths and areas for improvement.
Written examination	• a task undertaken individually, under formal supervised conditions to gather evidence about student achievement in relation to knowledge, understanding and skills at a point in time, for example a half-yearly, yearly or trial HSC examination • a task which may include one or more unseen questions or items, assessing a range of outcomes and content.

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Mathematics Extension 1 Draft Assessment Requirements

The draft guidelines for school-based assessment provide specific advice about the number of formal assessment tasks, course components and weightings, and the nature of task types to be administered in Year 11 and Year 12.

The components and weightings for Year 11 and Year 12 are mandatory.

Year 11

- There will be 3 formal assessment tasks
- The maximum weighting for each formal assessment task is 40%
- One task must be an assignment or investigation-style task with a weighting of 20–30%
- Students undertaking internal Mathematics Extension 1 assessment are not required to undertake internal Mathematics assessment.

Component	**Weighting %**
Knowledge, understanding and communication	50
Problem solving, reasoning and justification	50
	100

Year 12

- There will be no more than 4 formal assessment tasks
- The maximum weighting for each formal assessment task is 40%
- One task may be a formal written examination, eg a trial HSC, with a maximum weighting of 25%
- One task must be an assignment or investigation-style task with a weighting of 20–30%
- Students undertaking internal Mathematics Extension 1 assessment are not required to undertake internal Mathematics assessment.

Component	**Weighting %**
Knowledge, understanding and communication	50
Problem solving, reasoning and justification	50
	100

Mathematics Extension 1 Draft Examination Specifications
Option 1

Sections
Section – Written responses A variety of short answer questions to extended responses, with the possibility of a number of parts

Changes from current examination specifications
The objective response section will be removed. This approach provides opportunity for inclusion of cross-strand application questions, and modelling and problem solving questions to enable students to demonstrate deep understanding, conceptual knowledge, higher-order thinking and reasoning.

Questions or parts of questions may be drawn from a range of syllabus outcomes and content.

25% of marks will be common with the Mathematics examination. However, there will be additional significant overlap of course material with Mathematics within the examination.
Preliminary/Year 11 material will not be assessed in the HSC examination.

Mathematics Extension 1 students are no longer required to sit the Mathematics examination in addition to the Mathematics Extension 1 examination.

Option 2

Sections
Section I – Objective responses
Section II – Written responses A variety of short answer questions to extended responses, with the possibility of a number of parts

Changes from current examination specifications
Questions or parts of questions may be drawn from a range of syllabus outcomes and content.

25% of marks will be common with the Mathematics examination. However, there will be additional significant overlap of course material with Mathematics within the examination.
Preliminary/Year 11 material will not be assessed in the HSC examination.

Mathematics Extension 1 students are no longer required to sit the Mathematics examination in addition to the Mathematics Extension 1 examination.

HSC examination specifications will be reviewed following the finalisation of the syllabuses.

Updated assessment and reporting advice will be provided when syllabuses are released.

The Assessment Certification Examination website will be updated to align with the syllabus implementation timeline.

CONTENT

For Kindergarten to Year 12, courses of study and educational programs are based on the outcomes and content of syllabuses. The content describes in more detail how the outcomes are to be interpreted and used, and the intended learning appropriate for each Year. In considering the intended learning, teachers will make decisions about the emphasis to be given to particular areas of content, and any adjustments required based on the needs, interests and abilities of their students.

The knowledge, understanding and skills described in the outcomes and content provide a sound basis for students to successfully transition to their selected post-school pathway.

LEARNING ACROSS THE CURRICULUM

for your information

NSW syllabuses provide a context within which to develop core skills, knowledge and understanding considered essential for the acquisition of effective, higher-order thinking skills that underpin successful participation in further education, work and everyday life including problem-solving, collaboration, self-management, communication and information technology skills.

BOSTES has described learning across the curriculum areas that are to be included in syllabuses. In Stage 6 syllabuses, the identified areas will be embedded in the descriptions of content and identified by icons. Learning across the curriculum content, including the cross-curriculum priorities and general capabilities, assists students to achieve the broad learning outcomes defined in the BOSTES *Statement of Equity Principles*, the *Melbourne Declaration on Educational Goals for Young Australians* (December 2008) and in the Australian Government's *Core Skills for Work Developmental Framework* (2013).

Knowledge, understanding, skills, values and attitudes derived from the learning across the curriculum areas will be included in BOSTES syllabuses, while ensuring that subject integrity is maintained.

Cross-curriculum priorities enable students to develop understanding about and address the contemporary issues they face.

The cross-curriculum priorities are:
- Aboriginal and Torres Strait Islander histories and cultures
- Asia and Australia's engagement with Asia
- Sustainability

General capabilities encompass the knowledge, skills, attitudes and behaviours to assist students to live and work successfully in the 21st century.

The general capabilities are:
- Critical and creative thinking
- Ethical understanding
- Information and communication technology capability
- Intercultural understanding
- Literacy
- Numeracy
- Personal and social capability

BOSTES syllabuses include other areas identified as important learning for all students:
- Civics and citizenship
- Difference and diversity
- Work and enterprise

consult

Aboriginal and Torres Strait Islander histories and cultures

Through application and modelling across the strands of the syllabus, students can experience the relevance of mathematics in Aboriginal and Torres Strait Islander histories and cultures. Opportunities are provided to connect mathematics with Aboriginal and Torres Strait Islander peoples' cultural, linguistic and historical experiences. The narrative of the development of Mathematics and its integration with cultural development can be explored in the context of some topics. Through the evaluation of statistical data where appropriate, students can deepen their understanding of the lives of Aboriginal and Torres Strait Islander peoples.

When planning and programming content relating to Aboriginal and Torres Strait Islander histories and cultures teachers are encouraged to consider involving local Aboriginal communities and/or appropriate knowledge holders in determining suitable resources, or to use Aboriginal or Torres Strait Islander authored or endorsed publications.

Asia and Australia's engagement with Asia

Students have the opportunity to learn about the understandings and application of mathematics in Asia and the way mathematicians from Asia continue to contribute to the ongoing development of mathematics. By drawing on knowledge of and examples from the Asia region, such as calculation, money, art, architecture, design and travel, students can develop mathematical understanding in fields such as number, patterns, measurement, symmetry and statistics. Through the evaluation of statistical data, students can examine issues pertinent to the Asia region.

Sustainability

Mathematics provides a foundation for the exploration of issues of sustainability. Students have the opportunity to learn about the mathematics underlying topics in sustainability such as: energy use and how to reduce it; alternative energy with solar cells and wind turbines; climate change and mathematical modelling. Through measurement and the reasoned use of data students can measure and evaluate sustainability changes over time and develop a deeper appreciation of the world around them. Mathematical knowledge, understanding and skills are necessary to monitor and quantify both the impact of human activity on ecosystems and changes to conditions in the biosphere.

Critical and creative thinking

Critical and creative thinking are key to the development of mathematical understanding. Mathematical reasoning and logical thought are fundamental elements of critical and creative thinking. Students are encouraged to be critical thinkers when justifying their choice of a calculation strategy or identifying relevant questions during an investigation. They are encouraged to look for alternative ways to approach mathematical problems; for example, identifying when a problem is similar to a previous one, drawing diagrams or simplifying a problem to control some variables. Students are encouraged to be creative in their approach to solving new problems, combining the skills and knowledge they have acquired in their study of a number of different topics in a new context.

Ethical understanding

Mathematics makes a clear distinction between basic principles and the deductions made from them or their consequences in different circumstances. Students have opportunities to explore, develop and apply ethical understanding to mathematics in a range of contexts. Examples include: collecting, displaying and interpreting data; examining selective use of data by individuals and organisations; detecting and eliminating bias in the reporting of information; exploring the importance of fair comparison and interrogating financial claims and sources.

Information and communication technology capability

Mathematics provides opportunities for students to develop ICT capacity when students investigate; create and communicate mathematical ideas and concepts using fast, automated, interactive and multimodal technologies. Students can use their ICT capability to perform calculations; draw graphs; collect, manage, analyse and interpret data; share and exchange information and ideas; and investigate and model concepts and relationships. Digital technologies, such as calculators, spreadsheets, dynamic geometry software, graphing software and computer algebra software, can engage students and promote understanding of key concepts.

Intercultural understanding

Students have opportunities to understand that mathematical expressions use universal symbols, while mathematical knowledge has its origin in many cultures. Students realise that proficiencies such as understanding, fluency, reasoning and problem-solving are not culture- or language-specific, but that mathematical reasoning and understanding can find different expression in different cultures and languages. The curriculum provides contexts for exploring mathematical problems from a range of cultural perspectives and within diverse cultural contexts. Students can apply mathematical thinking to identify and resolve issues related to living with diversity.

Literacy

Literacy is used throughout mathematics to understand and interpret word problems and instructions that contain the particular language features of mathematics. Students learn the vocabulary associated with mathematics, including synonyms, technical terminology, passive voice and common words with specific meanings in a mathematical context. Literacy is used to pose and answer questions, engage in mathematical problem-solving and to discuss, produce and explain solutions. There are opportunities for students to develop the ability to create and interpret a range of texts typical of mathematics ranging from graphs to complex data displays.

Numeracy

Mathematics has a central role in the development of numeracy in a manner that is more explicit and foregrounded than is the case in other learning areas. It is related to a high proportion of the content. Consequently, this particular general capability is not tagged in the syllabus.

Numeracy involves drawing on knowledge of particular contexts and circumstances in deciding when to use mathematics, choosing the mathematics to use and critically evaluating its use. To be numerate is to use mathematics effectively to meet the general demands of life at home, in work, and for participation in community and civic life. It is therefore important that the mathematics curriculum provides the opportunity to apply mathematical understanding and skills in context, in other learning areas and in real-world contexts.

Personal and social capability

Students develop personal and social competence as they learn to understand and manage themselves, their relationships and their lives more effectively. Mathematics enhances the development of students' personal and social capabilities by providing opportunities for initiative taking, decision-making, communicating their processes and findings, and working independently and collaboratively in the mathematics classroom. Students have the opportunity to apply mathematical skills in a range of personal and social contexts. This may be through activities that relate learning to their own lives and communities, such as time management, budgeting and financial management, and understanding statistics in everyday contexts.

Civics and citizenship

Mathematics has an important role in civics and citizenship education because it has the potential to help us understand our society and our role in shaping it. The role of mathematics in society has expanded significantly in recent decades as almost all aspects of modern-day life are quantified. Through modelling reality with mathematics and then manipulating the mathematics in order to understand and/or predict reality, students have the opportunity to learn mathematical knowledge, skills and understanding that are essential for active participation in the world in which we live.

Difference and diversity

Students make sense of and construct mathematical ideas in different ways, drawing upon their own unique experiences in life and mathematics learning. By valuing students' diversity of ideas, teachers foster students' efficacy in learning mathematics.

Work and enterprise

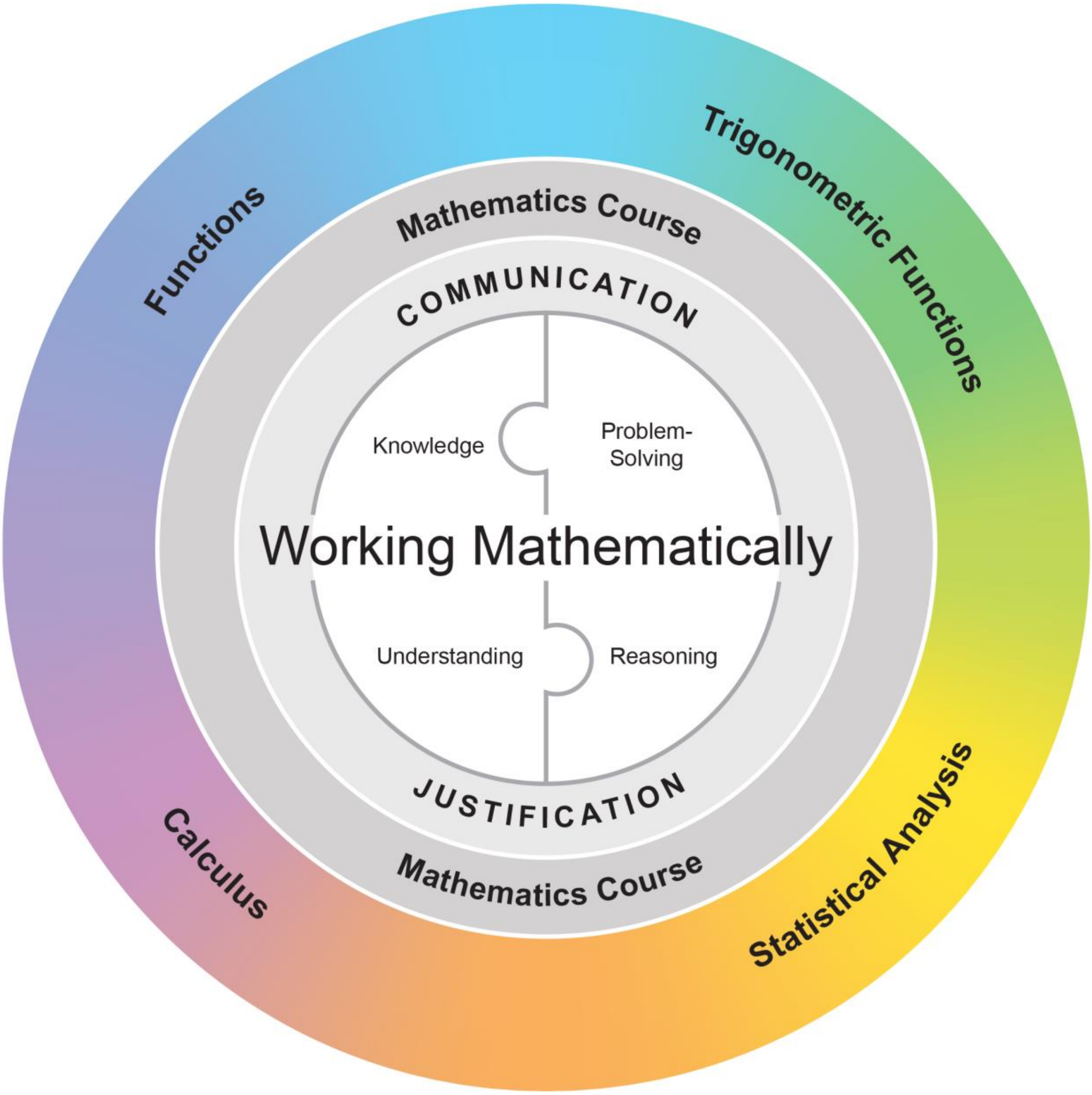
Students develop work and enterprise knowledge, understanding and skills through their study of mathematics in a work-related context. Students are encouraged to select and apply appropriate mathematical techniques and problem-solving strategies through work-related experiences in the financial mathematics and statistical analysis strands. This allows them to make informed financial decisions by selecting and analysing relevant information.

ORGANISATION OF CONTENT

for your information

The following provides a diagrammatic representation of the relationships between syllabus content.

consult



WORKING MATHEMATICALLY

Working Mathematically is integral to the learning process in Mathematics. It provides students with the opportunity to engage in genuine mathematical activity and develop the skills to become flexible, critical and creative users of mathematics. In this syllabus, Working Mathematically is represented through two key components: *Knowledge, Understanding and Communication* and *Problem-Solving, Reasoning and Justification*. Together these form the focus of the syllabus, and the components of assessment.

Knowledge, Understanding and Communication
Students make connections between related concepts and progressively apply familiar mathematical concepts and experiences to develop new ideas. They develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recall factual knowledge and concepts readily. Students communicate their chosen methods and efficiently calculated solutions, and develop an understanding of the relationship between the 'why' and the 'how' of mathematics. They represent concepts in different ways, identify commonalities and differences between aspects of content, communicate their thinking mathematically, and interpret mathematical information.

Problem-Solving, Reasoning and Justification
Students develop their ability to interpret, formulate, model and analyse identified problems and challenging situations. They describe, represent and explain mathematical situations, concepts, methods and solutions to problems using appropriate language, terminology, tables, diagrams, graphs, symbols, notation and conventions. They apply mathematical reasoning when they explain their thinking, deduce and justify strategies used and conclusions reached, adapt the known to the unknown, transfer learning from one context to another, prove that something is true or false, and compare and contrast related ideas and explain choices. Their communication is powerful, logical, concise and precise.

Both components and hence, Working Mathematically, are evident across the range of syllabus strands, objectives and outcomes. Teachers extend students' levels of proficiency in Working Mathematically by creating opportunities for development through the learning experiences that they design.

MATHEMATICS EXTENSION 1 YEAR 11 COURSE CONTENT

consult

STRAND: TRIGONOMETRIC FUNCTIONS

OUTCOMES

A student:

> applies concepts and techniques of inverse trigonometric functions in the solutions of problems ME11-2

> uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6

> communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

STRAND FOCUS

Trigonometric Functions involves the exploration of inverse trigonometric functions over restricted domains and their behaviour in both an algebraic and graphical form.

Knowledge of trigonometric functions enables the solving of practical problems involving the finding of inverse trigonometric functions.

The study of trigonometric functions is important in developing students' understanding of the links between algebraic and graphical representations and how this can be applied to solve practical problems.

TOPICS

ME-T1 Inverse Trigonometric Functions and Graphs

TRIGONOMETRIC FUNCTIONS

ME-T1 INVERSE TRIGONOMETRIC FUNCTIONS AND GRAPHS

OUTCOMES

A student:

> applies concepts and techniques of inverse trigonometric functions in the solutions of problems ME11-2

> uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6

> communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

TOPIC FOCUS

The principal focus of this topic is for students to determine and work with the inverse trigonometric functions.

Students develop knowledge of inverse function graphs and their relationships with their algebraic representation.

CONTENT

Students:

- define formally the inverse trigonometric functions, for example $g(x) = sin^{-1}x$, and sketch these functions and transformations of them (ACMSM095, ACMSM096, ACMSM097)
  - use the convention of restricting the domain of $sin\ x$ to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, $cos\ x$ to $0 \le x \le \pi$ and $tan\ x$ to $-\frac{\pi}{2} < x < \frac{\pi}{2}$.
- use the relationships $sin(sin^{-1}x) = x$ and $sin^{-1}(sin\ x) = x, cos(cos^{-1}x) = x$ and $cos^{-1}(cos\ x) = x,$ and $tan(tan^{-1}x) = x$ and $tan^{-1}(tan\ x) = x$ where appropriate
  - state the values of x for which the relationship $sin(sin^{-1}x) = x$ is valid and for which the relationship $sin^{-1}(sin\ x) = x$ is valid
  - state the values of x for which the relationship $cos(cos^{-1}x) = x$ is valid and for which the relationship $cos^{-1}(cos\ x) = x$ is valid
  - state the values of x for which the relationship $tan(tan^{-1}x) = x$ is valid and for which the relationship $tan^{-1}(tan\ x) = x$ is valid

STRAND: FUNCTIONS

OUTCOMES

A student:

> uses algebraic and graphical concepts in the solutions and sketching of functions and their inverses ME11-1

> applies algebraic techniques to find solutions of equations and inequalities ME11-3

> uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6

> communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

STRAND FOCUS

Functions involves the extended exploration of functions to include inequalities, absolute values and inverse functions as well as the use of algebraic concepts and techniques to describe and interpret relationships of and between changing quantities.

Knowledge of functions enables students to discover connections between abstract algebra, solutions of equations and their various graphical representations.

Study of functions is important in developing students' ability to find connections, communicate concisely, use algebraic techniques and manipulations to describe and solve problems, and to predict future outcomes.

TOPICS

ME-F1 Further Work with Functions
ME-F2 Polynomials

FUNCTIONS

ME-F1 FURTHER WORK WITH FUNCTIONS

OUTCOMES

A student:

> uses algebraic and graphical concepts in the solutions and sketching of functions and their inverses ME11-1

> applies algebraic techniques to find solutions of equations and inequalities ME11-3

> uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6

> communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

TOPIC FOCUS

The principal focus of this topic is to further explore functions in a variety of contexts.

Students develop appreciation for methods to identify solutions to equations both algebraically and graphically.

CONTENT

F1.1: Inequalities

Students:

- solve linear <u>inequalities</u> and <u>quadratic inequalities</u>
- solve inequalities involving fractions, including those with the unknown in the denominator
- solve absolute value inequalities of the form $|ax \pm b| > k$ or $|ax \pm b| < k$

F1.2: Inverse Functions

Students:

- find the equation of the inverse of a function (ACMSM095)
  - use the notation $y = f^{-1}(x)$for the <u>inverse function</u> of $y = f(x)$, including any domain restrictions
  - restrict the domain of a function to obtain a new function that is one-to-one
- determine the relationship between the graph of a function, $y = f(x)$,and the graph of its inverse,$x = f(y)$ (ACMSM096)
- solve problems based on the relationship between a function and its inverse function using algebraic or graphical techniques

F1.3: Graphical Relationships

Students:

- determine the relationship between the graph of $y = f(x)$(where $f(x)$ is a polynomial of at most degree 3) and:
  - the graph of $y = \frac{1}{f(x)}$(ACMSM099)
  - the graph of $y = \sqrt{f(x)}$ and $y^2 = f(x)$
  - the graphs of $y = |f(x)|$ and $y = f(|x|)$ (ACMSM099)
- determine the relationship between the graphs of $y = f(x)$and $y = g(x)$ (where $f(x)$and $g(x)$are polynomials of at most degree 3) and:

  - the graph of $y = f(x) + g(x)$
  - the graph of $y = f(x)g(x)$
- apply knowledge of graphical relationships to solve problems in practical and abstract contexts **M**
- utilise an initial set of equations and parameters to explore the effects of transformations. This investigation could culminate in the creation of a design for a product such as a pattern on a tile **E**

FUNCTIONS

ME-F2 POLYNOMIALS

OUTCOMES

A student:

> uses algebraic and graphical concepts in the solutions and sketching of functions and their inverses ME11-1

> applies algebraic techniques to find solutions of equations and inequalities ME11-3

> uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6

> communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

TOPIC FOCUS

The principal focus of this topic is to explore the algebraic structure of polynomials.

Students develop knowledge, understanding and skills to manipulate and solve polynomial equations.

CONTENT

F2.1: Remainder and Factor Theorems

Students:

- use division of polynomials to express $P(x)$ in the form $P(x) = A(x).Q(x) + R(x)$ where $deg\ R(x) < deg\ A(x)$ and $A(x)$ is a linear or quadratic divisor, $Q(x)$ the quotient and $R(x)$ the remainder
  - describe the process of division using the terms: dividend, divisor, quotient, remainder
  - perform long division with polynomials
- prove and apply the factor theorem and the remainder theorem for polynomials and hence solve simple polynomial equations (ACMSM089, ACMSM091)

F2.2: Sums and Products of Roots of Polynomials

Students:

- solve problems using the relationship between the roots and coefficients of quadratic, cubic and quartic equations **M**
  - consider quadratic, cubic and quartic equations, and derive formulae as appropriate for the sums and products of roots in terms of the coefficients
  - determine the multiplicity of a root of a polynomial equation
  - prove that if a polynomial equation of the form $P(x) = 0$ has a root of multiplicity $r > 1$, then $P'(x) = 0$ has a root of multiplicity $r - 1$
  - understand the link between the root of a polynomial equation and the zero on the graph of the related polynomial function

STRAND: CALCULUS

OUTCOMES

A student:

> applies their understanding of the concept of a derivative to solve related rates of change problems ME11-4

> uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6

> communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

STRAND FOCUS

Calculus involves the development of differential techniques to situations involving related rates of change and investigating the connectedness of a variety of rates of change due to the physical nature of quantities concerned.

Knowledge of calculus enables an understanding of the concept of related rates of change and how many practical situations could be described and problems solved using these techniques.

Study of calculus is important in developing students' knowledge and understanding of related rates of change, manipulating algebraic expressions and justifying their solutions.

TOPICS

ME-C1 Related Rates of Change

CALCULUS

ME-C1 RELATED RATES OF CHANGE

OUTCOMES

A student:

- > applies their understanding of the concept of a derivative to solve related rates of change problems ME11-4
- > uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
- > communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

TOPIC FOCUS

The principal focus of this topic is for students to solve problems involving the chain rule, in particular applied to related rates of change.

Students develop the ability to communicate the relationship between variables and rates and hence apply this to situations involving related rates of change.

CONTENT

Students:

- understand the notion of the composition of functions, and use the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, for determining the derivatives of composite functions (ACMMM105)
- solve problems involving related rates of change as instances of the chain rule (ACMSM129) **M**
- develop models of contexts where a rate of change of a formula can be expressed as a rate of change of a composition of two functions, to which the chain rule can be applied

STRAND: STATISTICAL ANALYSIS

OUTCOMES

A student:

> uses concepts of permutations and combinations to solve problems involving counting or ordering ME11-5
> uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
> communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

STRAND FOCUS

Statistical Analysis involves counting and ordering as well as exploring values, patterns, symmetry and methods to generalise and predict outcomes.

A knowledge of statistical analysis enables careful interpretation of situations and awareness of all contributing factors when presented with information by third parties, including possible misrepresentation of information.

Study of statistical analysis is important in developing students' ability to generalise situations, explore patterns and ensure that they have considered all outcomes.

TOPICS

ME-S1 Permutations and Combinations

STATISTICAL ANALYSIS

ME-S1 PERMUTATIONS AND COMBINATIONS

OUTCOMES

A student:

> uses concepts of permutations and combinations to solve problems involving counting or ordering ME11-5

> uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6

> communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7

TOPIC FOCUS

The principal focus of this topic is to develop students' understanding of permutations and combinations and its link to the binomial coefficients.

Students develop appreciation of order and counting techniques in restricted and unrestricted situations.

CONTENT

Students:

- use the notation ${}^nP_r$ and the formula $\frac{n!}{(n-r)!}$ for the number of permutations of r objects taken from a set of n distinct objects, and apply this to solve problems (ACMSM001, ACMSM003)
  - understand that the number of arrangements or permutations of n distinct objects is $n!$(ACMSM002)
  - solve problems with and without restrictions (ACMSM004)
  - solve problems and prove results using the pigeon-hole principle (ACMSM006)
- use the notation $\binom{n}{r}$ or ${}^nC_r$ and the formula $\binom{n}{r} = \frac{{}^nP_r}{r!} = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects taken from a set of n distinct objects, and apply this to solve problems (ACMMM045, ACMSM007, ACMSM008)
  - understand the notion of a combination as an unordered set of r objects selected from a set of n distinct objects (ACMMM044)
  - solve problems involving combinations
- use simple identities associated with Pascal's triangle and link to binomial coefficients (ACMSM009)
  - recognise the numbers $\binom{n}{r}$ as binomial coefficients, (coefficients in the expansion of $(x+y)^n$) (ACMMM047)
  - expand $(x+y)^n$ for small positive integers n (ACMMM046)
  - use Pascal's triangle and its properties (ACMMM048)
- solve practical problems involving permutations and combinations, including those involving probability **M**
- investigate alternate contexts involving patterns and symmetry such as Fibonacci's sequence, and hence apply Fibonacci's sequence to determine the golden ratio and link trigonometry and Phi **E**

MATHEMATICS EXTENSION 1 YEAR 12 COURSE CONTENT

consult

STRAND: TRIGONOMETRIC FUNCTIONS

OUTCOMES

A student:

> applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations ME12-2

> chooses and uses appropriate technology to solve problems in a range of contexts, and applies critical thinking to recognise appropriate times for such use ME12-6

> evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

STRAND FOCUS

Trigonometric Functions involves appreciation of both algebraic and geometric methods to solve trigonometric problems.

Knowledge of trigonometric functions enables the capacity to manipulate trigonometric expressions to prove identities and solve equations.

Study of trigonometric functions is important in developing students' understanding of the links between algebraic and graphical representations and how this can be applied to solve practical problems.

TOPICS

ME-T2 Trigonometric Equations

TRIGONOMETRIC FUNCTIONS

ME-T2 TRIGONOMETRIC EQUATIONS

OUTCOMES

A student:

> applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations ME12-2

> chooses and uses appropriate technology to solve problems in a range of contexts, and applies critical thinking to recognise appropriate times for such use ME12-6

> evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

TOPIC FOCUS

The principal focus of this topic is to consolidate and extend students' knowledge in relation to solving trigonometric equations and their application to practical situations.

Students develop complex algebraic manipulative skills and fluency in applying trigonometric knowledge in a variety of situations.

CONTENT

T2.1: Further Trigonometric Identities

Students:

- derive and use the expansions for trigonometric functions $sin(A \pm B)$, $cos(A \pm B)$ and $tan(A \pm B)$, including the results for double angles (ACMSM044, ACMMM041)
  - derive and use the products as sums and differences (ACMSM047)
  - derive and use the half-angle results expressions for $sin\ \theta$, $cos\ \theta$ and $tan\ \theta$in terms of $tan\ \frac{\theta}{2}$

T2.2: Solving Trigonometric Equations

Students:

- convert expressions of the form $acos\ x + bsin\ x$ to $Rcos(x \pm \alpha)$ or $Rsin(x \pm \alpha)$ and apply these to solve equations of the form $acos\ x + bsin\ x\ =\ c$ and/or sketch graphs, and solve related problems (ACMSM048)
- solve trigonometric equations requiring factorising and/or the application of sums and differences of angles, double angles and half angles (ACMSM049)
- using digital technology or otherwise, solve trigonometric equations and interpret solutions in context

STRAND: CALCULUS

OUTCOMES

A student:

> applies calculus techniques to model and solve problems ME12-1
> integrates rational functions and uses techniques for the determination of definite integrals and areas and the calculation of volumes of revolution ME12-5
> chooses and uses appropriate technology to solve problems in a range of contexts, and applies critical thinking to recognise appropriate times for such use ME12-6
> evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

STRAND FOCUS

Calculus involves the development of analytic and numeric integration techniques and the extension of this to solving problems involving exponential growth and decay and geometric applications to find the volume of solids of revolution.

Knowledge of calculus enables understanding of the concept of differentiation and integration techniques and the use of these in solving practical problems.

Study of calculus is important in developing students’ knowledge and understanding of the links between a function, the derivative function, the anti-derivative function and the volume of a solid created by rotating that function around an axis.

TOPICS

ME-C2 Further Calculus Skills
ME-C3 Exponential Growth and Decay
ME-C4 Volumes of Solids of Revolution

CALCULUS

ME-C2 FURTHER CALCULUS SKILLS

OUTCOMES

A student:

> applies calculus techniques to model and solve problems ME12-1

> uses integral calculus in the solution of problems including volumes of solids of revolution requiring identification and use of appropriate substitutions, partial fractions and integration by parts ME12-4

> chooses and uses appropriate technology to solve problems in a range of contexts, and applies critical thinking to recognise appropriate times for such use ME12-6

> evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

TOPIC FOCUS

The principal focus of this topic is to further develop students' knowledge, understanding and skills relating to differentiation and integration techniques.

Students develop awareness of the connections between this strand and others, and the fluency that can be obtained in the utilisation of calculus techniques.

CONTENT

Students:

- use the relationship $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to find derivatives of inverse functions
  - determine the derivatives of $sin^{-1}(\frac{x}{a})$, $cos^{-1}(\frac{x}{a})$ and $tan^{-1}(\frac{x}{a})$ by using the properties of inverse functions
- find and evaluate integrals using the method of <u>integration by substitution</u>
  - use substitution $u = g(x)$to integrate expressions of the form $f(g(x))g'(x)$ (ACMSM117)
- solve problems involving $\int sin^2nx\, dx$ and/or $\int cos^2nx\, dx$ (ACMSM116)
  - use the expansions of $cos\, 2x$ to integrate sin^2nx and cos^2nx

CALCULUS

ME-C3 EXPONENTIAL GROWTH AND DECAY

OUTCOMES

A student:

> applies calculus techniques to model and solve problems ME12-1

> chooses and uses appropriate technology to solve problems in a range of contexts, and applies critical thinking to recognise appropriate times for such use ME12-6

> evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

TOPIC FOCUS

The principal focus of this topic is to introduce students to the concept of modelling exponential growth and decay problems mathematically.

Students develop awareness of the connections between this strand and others, and the fluency that can be obtained in the utilisation of calculus techniques.

CONTENT

Students:

- construct, analyse and solve an exponential model to solve a practical growth or decay problem of the form $f(t) = P + Ae^{kt}$(eg population growth, radio-active decay or depreciation) (ACMMM064, ACMMM066, ACMMM067) **M**
  - describe the rate of change of a population N by $\frac{dN}{dt} = kN$, where k is the population growth constant
  - demonstrate that the function $N(t) = Ae^{kt}$ satisfies the relationship $\frac{dN}{dt} = kN$, with A being the initial value of N
- construct, analyse and solve an exponential model to solve a practical growth or decay problem of the form $f(t) = P + Ae^{kt}$(eg 'Newton's Law of Cooling' or an ecosystem with a natural 'carrying capacity') (ACMMM064, ACMMM066, ACMMM067) **M**
  - describe with the equation $\frac{dN}{dt} = k(N - P)$, the rate of change in a quantity N which varies directly with the difference between N and a constant, P
  - verify by substitution that a solution to the differential equation $\frac{dN}{dt} = k(N - P)$ is $N = P$, and that $N = P + Ae^{kt}$ is another solution, for an arbitrary constant A
  - determine that whenever $k < 0$, the quantity N tends to the limit P as $t \to \infty$, irrespective of the initial conditions

CALCULUS

ME-C4 VOLUME OF SOLIDS OF REVOLUTION

OUTCOMES

A student:

> applies calculus techniques to model and solve problems ME12-1

> uses integral calculus in the solution of problems including volumes of solids of revolution requiring identification and use of appropriate substitutions, partial fractions and integration by parts ME12-4

> chooses and uses appropriate technology to solve problems in a range of contexts, and applies critical thinking to recognise appropriate times for such use ME12-6

> evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

TOPIC FOCUS

The principal focus of this topic is to determine the volume of a solid where the boundary is formed by rotating the arc of a simple function around either axis to form a solid of revolution.

Students develop awareness of the use of calculus to solve practical problems involving solids.

CONTENT

Students:

- sketch and calculate the volume of a solid of revolution whose boundary is formed by rotating an arc of a simple function about the x-axis or y-axis, using digital technology or otherwise (ACMSM125)
  - derive and use the formula $V = \pi \int_a^b [f(x)]^2 dx$
  - determine the volumes of solids of revolution about either axis that are formed by at most two different simple functions in both real-life and abstract contexts **M**
- design an appropriate set of equations to be utilised to create a wine glass or similar geometric figure on a production line **E**

STRAND: STATISTICAL ANALYSIS

OUTCOMES

A student:

> applies appropriate statistical processes to present, analyse and interpret data ME12-5

> chooses and uses appropriate technology to solve problems in a range of contexts, and applies critical thinking to recognise appropriate times for such use ME12-6

> evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

STRAND FOCUS

Statistical Analysis involves exploring the reliability of results based on sampling methodologies and estimates.

Knowledge of statistical analysis enables careful interpretation of situations and awareness of all contributing factors when presented with information by third parties, including possible misrepresentation of information.

Study of statistical analysis is important in developing students' ability to describe the level of reliability that can be applied to predict future outcomes, and an appreciation of how conclusions drawn from data can be used to inform decisions made by groups such as scientific investigators, business people and policy-makers.

TOPICS

ME-S2 Sampling and Estimates

STATISTICAL ANALYSIS

ME-S2 SAMPLING AND ESTIMATES

OUTCOMES

A student:

> applies appropriate statistical processes to present, analyse and interpret data ME12-5

> chooses and uses appropriate technology to solve problems in a range of contexts, and applies critical thinking to recognise appropriate times for such use ME12-6

> evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

TOPIC FOCUS

The principal focus of this topic is to estimate an unknown parameter associated with a population using a sample of data drawn from that population.

Students develop appreciation of the importance of sample selection and size.

CONTENT

Students:

- use graphical displays of simulated data to investigate the variability of random samples from various types of distributions, including uniform (discrete and continuous), normal and Bernoulli (ACMMM173) **M**
  - understand the concept of a random sample (ACMMM171)
  - discuss sources of bias in samples, and procedures to ensure randomness (ACMMM172) **M**
- calculate the mean p and standard deviation $\sqrt{\frac{p(1-p)}{n}}$ of the distribution of the sample proportion $\hat{p}$ (ACMMM174)
  - understand the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples
- use the approximate confidence interval $\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$, as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution (ACMMM178)
  - explore the effect of the level of confidence and sample size on the width of the confidence interval
  - understand the concept of a confidence interval or interval estimate for a population proportion, p (ACMMM177)
  - examine the approximate normality of the distribution of $\hat{p}$ for large samples (ACMMM175)
  - simulate repeated random sampling and recognise its strengths and weaknesses in illustrating the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}-p}{\sqrt{(\hat{p}(1-\hat{p})/n}}$ where the closeness of the approximation depends on both n and p (ACMMM176) **M**
- define the approximate standard error $E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ and understand the trade-off between margin of error and level of confidence (ACMMM179)
- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain p (ACMMM180) **M**

GLOSSARY

for your information

The glossary explains terms that will assist teachers in the interpretation of the subject. The glossary will be based on the NSW Mathematics K–10 glossary and the Australian curriculum senior secondary years Specialist Mathematics glossary.

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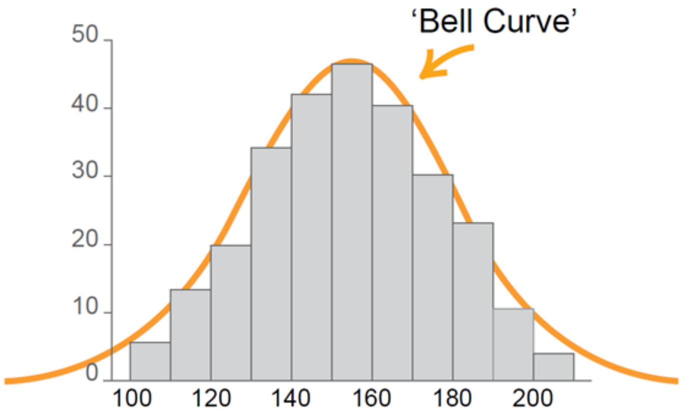
** Indicates new glossary terms*
**** Indicated glossary terms also in Mathematics*

Glossary term	Definition
acceleration*	Acceleration is the rate of change of velocity with respect to time. It is measured in terms of the length units and time units used for velocity. For example, km/h^2or m/s^2.
Bernoulli distribution*	A Bernoulli distribution is the probability distribution of a random variable which takes the value 1 with probability of p and the value 0 with probability $q = 1 - p$. A Bernoulli distribution is a special case of the binomial distribution, where $n = 1$.
Bernoulli random variable*	A Bernoulli random variable has two possible values, namely 0 and 1. The parameter associated with such a random variable is the probability p of obtaining a 1.
Bernoulli trial*	A Bernoulli trial is a chance experiment with possible outcomes, typically labelled ‘success’ and ‘failure’.
Cartesian equation*	A Cartesian equation is an equation in x and y describing a curve, so that the points of the curve are the points (x, y) such that x and y satisfy the equation. For example, $y = x^2 + 3x - 2$ or $x = y^2$. A Cartesian equation can be formed from two parametric equations by eliminating the parameter.
chain rule*	The chain rule relates the derivative of the composite of two functions to the functions and their derivatives. If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$ In other notation, if $h(x) = (f \circ g)(x)$ then $(f \circ g)'(x) = (f' \circ g)(x)g'(x)$ Or $\frac{du}{dx} = \frac{du}{dy} \times \frac{dy}{dx}$
combinations (selections)*	The number of selections of n objects taken r at a time (that is, the number of ways of selecting r objects out of n) is denoted by ${}^nC_r$ or $\binom{n}{r}$ and is equal to $\frac{n!}{r!(n-r)!}$

Glossary term	Definition
composition of functions*	When two functions are composed, they are performed one after the other. For example, the composite of f and g, acting on x, is written as $(f \circ g)(x)$ and means $f(g(x))$, with $g(x)$being performed first. If $y = g(x)$ and $z = f(y)$ for functions f and g, then z is a composite function of x. We write $z = (f \circ g)(x) = f(g(x))$. For example, $z = \sqrt{x^2 + 3}$ expresses z as a composite of the functions $f(y) = \sqrt{y}$ and $g(x) = x^2 + 3$.
confidence interval*	a confidence interval (CI) is a type of interval estimate of a population parameter. It is calculated to given a certain percentage confidence that a value lies within that interval.
dividend*	In a division sum, the dividend is the number being divided. For example, in the sum $6 \div 2 = 3$, the dividend is 6.
divisor*	In a division sum, the divisor is the number you are dividing by. For example, in the sum $6 \div 2 = 3$, the divisor is 2.
double angle formulae*	$sin\ 2A = 2\ sin\ A\ sin\ B$ $cos\ 2A = cos^2A - sin^2A$ $cos\ 2A = 2\ cos^2A - 1$ $cos\ 2A = 1 - 2\ sin^2A$ $tan\ 2A = \frac{2\ tan\ A}{1 - tan^2A}$
expansions for sums and differences of trig functions*	$sin\ (A \pm B) = sin\ A\ cos\ B \pm sin\ B\ cos\ A$ $cos\ (A \pm B) = cos\ A\ cos\ B \mp sin\ A\ sin\ B$ $tan\ (A \pm B) = \frac{tan\ A \pm tan\ B}{1 \mp tan\ A\ tan\ B}$
half angle results*	$\sin\theta = \frac{2t}{1 + t^2}$ $\cos\theta = \frac{1 - t^2}{1 + t^2}$ $tan\left(\frac{\theta}{2}\right) = \pm\frac{1 - cos\theta}{sin\theta}$ $\tan\theta = \frac{2t}{1 - t^2}$ Where $t = \tan\left(\frac{\theta}{2}\right)$

Glossary term	Definition
inequalities	An inequality is a statement that one number or algebraic expression is less than (or greater than) another. There are five types of inequalities: ● a is less than b is written $a < b$, ● a is greater than b is written $a > b$, ● a is less than or equal to b is written $a \leq b$, ● a is greater than or equal to b is written $a \geq b$, and ● a is not equal to b is written $a \neq b$.
interval estimate*	interval estimation is the use of sample data to calculate an interval of possible (or probable) values of an unknown population parameter.
inverse functions*	An inverse function is a function obtained by expressing the dependent variable of one function as the independent variable of another. f and g are said to be inverse functions if $f(g(x)) = x$ and $g(f(x)) = x$ for all x in the specified domain.
inverse trigonometric functions*	**The inverse sine function, $y = sin^{-1}x$** If the domain for the sine function is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ a one to one function is formed and so the inverse function exists. The inverse of this restricted sine function is denoted by sin^{-1}, or $arcsin$ and is defined by: $sin^{-1}: [-1,1] \rightarrow \Re, sin^{-1}x = y$ where $sin\, y = x, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **The inverse cosine function, $y = cos^{-1}x$** If the domain for the cosine function is restricted to the interval $[0, \pi]$ a one to one function is formed and so the inverse function exists. The inverse of this restricted cosine function is denoted by cos^{-1}, or $arccos$ and is defined by: $cos^{-1}: [-1,1] \rightarrow \Re, cos^{-1}x = y$ where $cos\, y = x, y \in [0, \pi]$ **The inverse tangent function, $y = tan^{-1}x$** If the domain for the tangent function is restricted to the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ a one to one function is formed and so an inverse function exists. The inverse of this restricted tangent function is denoted by tan^{-1}, or $arctan$ and is defined by: $tan^{-1}: \Re \rightarrow \Re, tan^{-1}x = y$ where $tan\, y = x, y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
multiplicity (of a root)	Given a polynomial $p(x)$, if $p(x) = (x - a)^k q(x)\,, q(a) \neq 0, k > 0$, then the root $x = a$ has multiplicity k.

Glossary term	Definition
normal distribution*	In cases where data tends to be around a central value with no bias left or right, it is said to have a "Normal Distribution" like this:  The Bell Curve is a Normal Distribution. The Normal Distribution has: ● mean = median = mode ● symmetry about the centre ● 50% of values less than the mean and 50% greater than the mean
Pascal's triangle*	Pascal's triangle is an arrangement of numbers where the nth row consists of the binomial coefficients ${}^nC_r$ or $\binom{n}{r}$ with the $r = 0, 1, \ldots, n$ 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1 In Pascal's triangle any term is the sum of the two terms 'above' it. For example, $10 = 4 + 6$.
parameter*	A parameter is a variable in terms of which other variables are expressed. For example, points on a unit circle may be expressed as $x = cos\ \theta, y = sin\ \theta$ where θ is a parameter.
parametric equations*	Parametric equations are used to describe the link between two variables with respect to a parameter. For example, a parameter t could be used to define $x = t^2$ and $y = 2t$. The parameter could then be eliminated to form a Cartesian equation, by rearranging the $y = 2t$ and then substituting it to form: $x = \left(\frac{y}{2}\right)^2$
period*	A function that repeats at regular intervals is said to be periodic, and the length of the smallest such interval is called the period.

Glossary term	Definition
periodic motion*	Periodic motion is motion repeated at equal intervals of time. Periodic motion is performed, for example, by a rocking chair, a vibrating tuning fork, a swing in motion or the Earth in its orbit around the Sun. It can be modelled using trigonometric functions.
permutations*	A permutation of n distinct objects is an arrangement of n objects where order is important. The number of arrangements of n objects is $n!$ The number of permutations of r distinct objects chosen from n distinct objects is denoted by ${}^nP_r$ and is equal to $$n(n-1)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$
pigeonhole principle*	The pigeonhole principle states that if n items are put into m containers, with $n > m$, then at least one container must contain more than one item.
polynomial functions*	A polynomial function is a finite linear combination of non-negative integer powers of the variable.
products as sums and differences*	$cos\ A\ cos\ B\ = \frac{1}{2}(cos\ (A-B) + cos\ (A+B))$ $sin\ A\ sin\ B\ = \frac{1}{2}(cos\ (A-B) - cos\ (A+B))$ $sin\ A\ cos\ B\ = \frac{1}{2}(sin\ (A+B) + sin\ (A-B))$ $cos\ A\ sin\ B\ = \frac{1}{2}(sin\ (A+B) + sin\ (A-B))$
projectile motion*	Projectile motion is a form of motion in which an object or particle (called a projectile) is projected or dropped near the earth's surface, and it moves along a curved path under the action of gravity only. The only force that acts on the object is gravity, which acts downwards to cause acceleration.
quadratic equations	A quadratic equation is a polynomial equation of degree 2.
quadratic function or relationship* **	A quadratic expression or function contains one or more of the terms in which the variable is raised to the second power, but no variable is raised to a higher power. Examples of quadratic expressions include $3x^2 + 7$ and $x^2 + 2xy + y^2 - 2x + y + 5$.
quadratic graph* **	A quadratic graph is a graph drawn to illustrate a quadratic function. Its shape is called a parabola.
quadratic inequality*	An inequality involving a quadratic expression or function.
quotient*	A quotient is the result of performing a division. If the division is not exact, the quotient is the integer part of the result. For example, 15 is divided by 2 is 7 remainder 1. The quotient is 7 and the remainder is 1.

Glossary term	Definition
range (of function)*	the range of a function is the set of "output" values for which the function is defined. In a graph, it is the y values.
rational functions*	A rational function is a function of the form $\frac{p(x)}{q(x)}$ when p, q are polynomials.
remainder*	A reminder is what remains after a division has been performed. For example, 15 is divided by 2 is 7 remainder 1. The quotient is 7 and the remainder is 1.
sample proportion*	The fraction of the sample of trials that were a success.
simple harmonic motion*	Simple harmonic motion is any motion where a restoring force is applied that is proportional to the displacement and in the opposite direction of that displacement. Or in other words, the more you pull it one way, the more it wants to return to the middle. Simple harmonic motion can be modelled with trigonometric (periodic) functions.
solid of revolution*	A solid of revolution is a solid figure whose surface is obtained by rotating a plane curve around some straight line (often the axis) that lies on the same plane
standard error*	The standard error has been described as an "absolute" quantity, equal to half the range of the confidence interval.
zero (of a graph)*	The zeros of a graph are the x −intercepts.